CHARACTERISTICS OF GRAVITATIONAL FLOW OF NONUNIFORMLY DISTRIBUTED THIN LIQUID LAYERS

E. G. Vorontsov and O. M. Yakhno

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It is shown experimentally as well as analytically that the gravitational flow of nonuniformly distributed thin liquid layers along the external surface of a vertical tube is a nonsteady film flow which later goes over into a jet flow.

The gravitational flow of a thin liquid film along a vertical surface is used in some technological devices [2, 8]. The nonuniformity of the distribution of the thin liquid layer along the sprayed perimeter results in a decrease of the heat and mass exchange coefficients and in the case of rupture of the film it leads to a sharp decrease of the area of the contact surfaces, of the wall and the liquid, liquid and gas; as a result these devices become inefficient [2]. Therefore the investigation of the nature of the flow of a thin layer of a viscous liquid with nonuniform initial distribution along the sprayed perimeter is of definite interest. An experimental investigation of such a flow was carried out on an experimental equipment in which the outer diameter of the sprayed vertical made of Kh18N10T steel was 25.4 mm and the length of

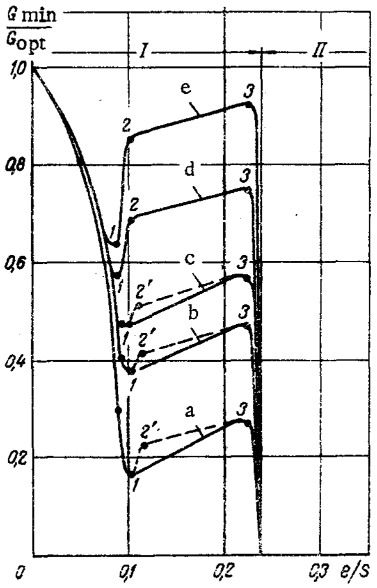

Fig. 1.

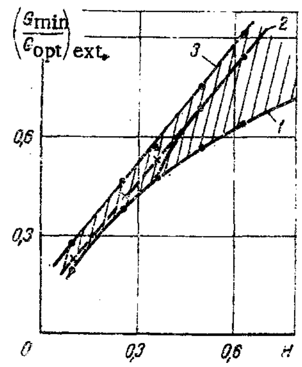

Fig. 2

Fig. 1. Dependence of the uniformity of wetting G_{min}/G_{opt} along the vertical tube on the relative eccentricity e/S of the distributor slit: curve a) H = 0.1 m, b) 0.26 m, c) 0.36 m, d) 0.5 m, e) 0.63 m; I) region of film flow. II) region of jet flow.

Fig. 2. Dependence of the uniformity of wetting (G_{min}/G_{opt}) on static thrust H of the liquid in the distributor: H, m.

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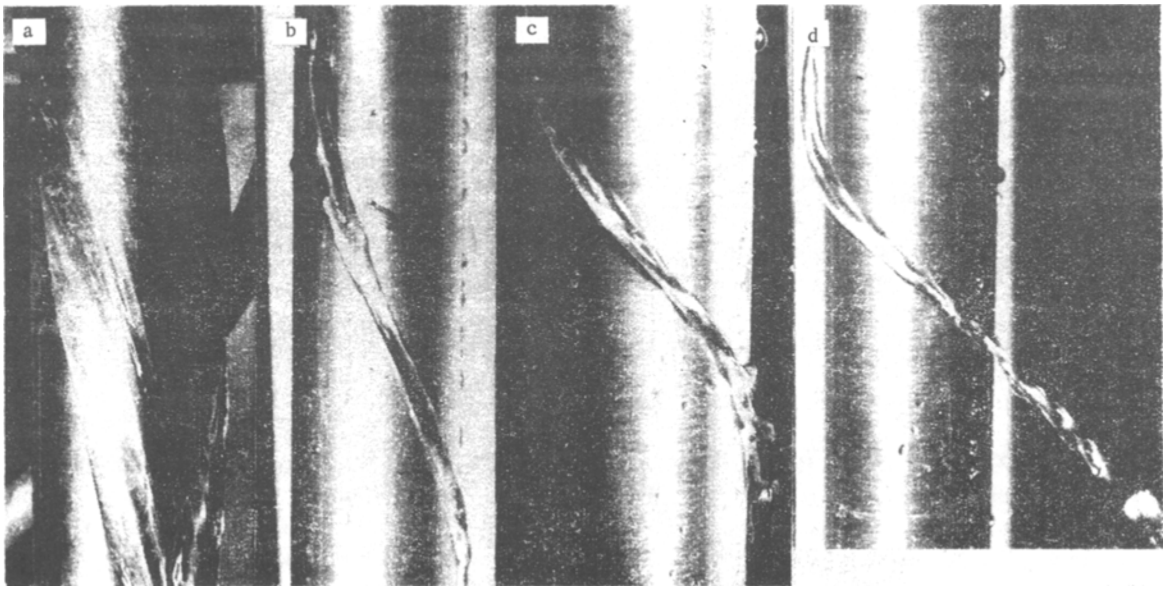

Fig. 3. Characteristics of jet flow in the presence of eccentric distribution of the liquid: a) rupture of liquid film and formation of jet; b) twisting of the jet distributed nonuniformly along the length; c) intensification of twisting of the jet and breaking of individual drops from it; d) separation of the jet from the surface of the tube and its fractionation into drops.

the tube was 710 mm. The spraying liquid (water) was distributed into a film at the upper end of the tube by a cylindrical slit formed by the outer diameter of the tube and the inner diameter of an interchangeable ring. The average width of the distributor slit was 0.5 mm. The interchangeable eccentric rings were made on a precision lathe with high accuracy; along the outer diameter the exit aperture of the frame of the distributor. The eccentricity e of the distributor slit varied in the range 0.025-0.322 mm and was measured by callibration thickness gauges with an accuracy up to 3%. The maximum relative errors in the measurements were 6% for e/S and 1.8% for the flow rate of water. The discharge G_{ve} of the wetting liquid and static thrust H in the distributor were measured. The lower end of the tube had eight teeth arranged uniformly over the perimeter; from each tooth the wetting liquid was fed into a separate tank, which made it possible to determine the minimum discharge of water (G_{min}) from 1/8 perimeter for the corresponding computation of its optimum discharge ($G_{opt} = G_{ve}/8$). The uniformity of the distribution of the liquid along the wetted perimeter was estimated by the ratio G_{min}/G_{opt}. The experimental dependence of the uniformity of wetting G_{min}/G_{opt} of the surface of the tube along the perimeter on the relative eccentricity e/S of the slit is shown in Fig. 1 for different static thrusts H of the liquid in the distributor.

For a strictly concentric distributor slit (e/S = 0) the uniformity of the distribution was $G_{min}/G_{opt} = 1$ for all static thrusts H (or for all flow rates of the liquid). The nature of the gravitational flow of a film in the case of its uniform distribution has been thoroughly investigated [2, 3, 8]. The presence of eccentricity for all flow rates leads to a decrease in the uniformity of distribution of the liquid over the perimeter of the tube especially in the range $0 < e/S \leq 0.09$-1. In this range of e/S a bulged liquid film emerges from the widest part of the slit. It runs vertically down along the generatrix of the tube and has a local convex wave front. A larger flow velocity is observed in this segment; the waves develop earlier and have larger amplitude and frequency. The flow occurs mainly under the force field consisting of gravitational, viscous frictional, and surface tension forces. In Fig. 1 this region is bounded by the extremal point 1.

A further increase of the eccentricity leads to an increase of excess body force [6] contributing to the development of a comparable tangential velocity component besides the axial, and hence results in a twisting of the flow. The flow gets accelerated and in respect of its structure as a function of the mean free path it can be represented in several segments. Close to the distributor, where the gravitational forces predominate, over a certain segment the flow has a weak structure. This can be called the segment of quasirectilinear flow. With the increase of the axial velocity component due to the excess body force the tangential velocity component increases leading to a curvature in the trajectories of the liquid particles and to a twisting of the flow around the tube. The bulged part of the film begins to get twisted with the axis of the tube as the twist axis. The main mass of the liquid gets collected in the twisted bulged part. The step of twisting decreases with the mean free path of the film. All this facilitates a relatively more uniform distribution of the mass of the liquid over the perimeter of the tube; the uniformity of distribution G_{min}/G_{opt} increases at first rapidly (up to point 2) and then somewhat slowly. In analogy with this perhaps one should expect the presence of extremal points 2' for small flow rates also ($H \leq 0.36$ m); then the form of the curves changes as shown by dashes in Fig. 1 (unfortunately additional experimental points could not be obtained in the range $0.1 < e/S < 0.225$). Naturally, the numerical values of extremal points 2 and 3 will also depend on the mean free path of the film.

The experimental dependence of the extremal values of the uniformity of wetting $(G_{min}/G_{opt})_{ext}$ on the static thrust H of the liquid in the distributor is shown in Fig. 2 for all eccentricities of the slit for the mean free path of the film equal to 710 mm. Curve 1, which describes the first extremal point, is described by the empirical equation

$$\left(\frac{G_{min}}{G_{opt}}\right)_{ext.1} \doteq 0.92 \cdot H^{0.71} \tag{1}$$

with a scatter of ±6%; curve 2 characterizing the obtained and expected points of the second extremum is described by the equation

$$\left(\frac{G_{min}}{G_{opt}}\right)_{ext.2} = 1.12 \cdot H + 0.1, \tag{2}$$

while curve 3 characterizing the third extemal point is well described by the equation

$$\left(\frac{G_{min}}{G_{opt}}\right)_{ext.3} = 1.17 \cdot H + 0.15. \tag{3}$$

The static thrust in the eccentric distributor can be determined analytically [1, 9] if we take into consideration the increase of the volumetric discharge of the liquid in the eccentric slit compared to an equivalent concentric slit [1]:

$$H = \frac{1}{2g}\left\{\frac{G_{ve}}{\mu f_0 n\left[1 + 1{,}5\left(\frac{e}{S}\right)^2\right]}\right\}^2. \tag{4}$$

Therefore with prespecified relative eccentricities e/S and volumetric discharge of the liquid G_{ve} formulas (1)-(3) enable us to compute the extremal values of the wetting uniformity. The shaded region between curve 1 and line 3 in Fig. 2 characterizes the amount of increase in the uniformity of wetting due to twisting of the flow for different H.

Thus the gravitational flow of a nonuniformly distributed thin liquid layer along the vertical surface of a tube occurs mainly under the action of the force due to gravity causing an acceleration $\bar{f}$ [5, 6]. However, forces due to the capillary pressure (P_σ) may also have a significant effect on the nature of the flow. The nonuniform distribution of the mass of the liquid along the wetted surface produces an excess body force which facilitates the twisting of the flow.

The magnitude of the excess body force, which changes depending on H and the ratio e/S, is determined by both coriolis and centrifugal forces. The twisted liquid flow in the film appearing as a result of this force may be regarded as a spiral flow with the pitch varying along the tube. The angular velocity $\bar{\omega}$ for this motion is proportional to $(e/S)^m$, where the power m (for external wetting of the tube $m > 0$, as a result of which $\bar{\omega}$ increases with the mean free path of the film; for internal wetting of the tube $m < 0$, which leads to a decrease of $\bar{\omega}$ and to quenching of the spiral motion due to viscous forces [4, 9]) is determined by the properties of the wetting liquid and wettability of the surface of the tube. The coefficient of proportionality $\varkappa$ depends on H and Reynolds number Re, i.e., $\varkappa = \varkappa(H, Re)$. Thus the angular velocity ω is

$$\omega = \varkappa(H, \mathrm{Re})\left(\frac{e}{S}\right)^m. \tag{5}$$

The velocity vector of the absolute motion of the liquid can be obtained from the formula

$$\bar{w}_a = \bar{w} + \bar{\omega} \times \bar{R}, \tag{6}$$

where $\bar{\omega}$ is the velocity vector of the translational motion of the flow along the tube and $\bar{\omega} \times \bar{R}$ is the velocity vector of the rotational motion.

In view of the condition of continuity of the flow the last formula shows that an increase of the angular velocity of the liquid results in a decrease of the translational motion and vice versa. However, as is evident from formula (5), an increase of e/S leads to an increase of ω, i.e. to an increase of the twisting of the flow. Therefore ω decreases with the increase of e/S.

Thus for $e/S > 0$ a complex field of body forces acts on the liquid flowing along the outer surface of the tube. If the gravitational forces are constant, the centrifugal forces increase (decrease for internal wetting) along the generatrix of the tube. Since the magnitude of the coriolis forces is proportional to the angular speed, while the magnitude of the centrifugal forces is proportional to the square of the angular speed, it follows that for small values of e/S the coriolis forces have a predominant effect on the twisting of the flow. In this case the pitch of the twisting is very large.

With the increase of the relative eccentricity e/S the centrifugal forces become decisive and the intensity of twisting increases. The main mass of the liquid collects in the bulged twisted part of the film and moves with a larger relative velocity than the thin untwisted part. For the relative eccentricity of the slit $e/S = 0.24$ the continuity of the film is disrupted, i.e. it gets ruptured. The film flow goes over into a jet flow. The angular velocity of the liquid increases, which leads to an increase of the inertial forces.

The characteristics of the jet flow in the presence of an eccentric distribution of the liquid are shown in Fig. 3. As the liquid flows out from the eccentric distributor slit the liquid film gets ruptured and a jet is formed (Fig. 3,a). The liquid collects in the twisted part in a jet distributed nonuniformly along the length and having approximately a cylindrical shape which twists around the tube (Fig. 3,b).

The trajectory of the jet is a spiral with variable pitch h(x) along the tube. The parametric equation of this trajectory can be written in the form

$$x = \frac{h(x)}{2\pi}\varphi; \quad y = R_{tr}\cos\varphi; \quad z = R_{tr}\sin\varphi. \tag{7}$$

The absolute velocity of the jet is

$$w_{a(y)} = \sqrt{w_{x(y)}^2 + w_{y(y)}^2 + w_{z(y)}^2}, \tag{8}$$

where the velocity components $w_{x(y)}$, $w_{y(y)}$, $w_{z(y)}$ are given by the formulas

$$|w_{x(y)}| = \frac{h(x)}{2\pi + \frac{dh}{dx}}\frac{d\varphi}{d\tau} = \frac{h(x)}{2\pi + \frac{dh}{dx}}\omega,$$

$$|w_{y(y)}| = R_{tr}\sin\varphi\frac{d\varphi}{d\tau} = (R_{tr}\sin\varphi)\,\omega,$$

$$|w_{z(y)}| = R_{tr}\cos\varphi\frac{d\varphi}{d\tau} = (R_{tr}\cos\varphi)\,\omega. \tag{9}$$

Then the absolute velocity of the jet can be given by the equation

$$w_{a(y)} = \omega\sqrt{R_{tr}^2 + \frac{h^2}{\left(2\pi + \frac{dh}{dx}\right)^2}}. \tag{10}$$

At the same time the jet gets twisted also around its own axis, which facilitates breaking-off of individual drops from it later (Fig. 3, c) and after that the separation of the entire jet from the surface of the tube and its fractionation into drops (Fig. 3,d). Additional special experiments are required for obtaining a sufficiently accurate quantitative relation, which would permit a detailed analytical description of the observed phenomena.

Therefore the flow of a viscous liquid along the outer surface of a vertical tube in the presence of eccentricity in the slit-type distributor represents a nonsteady film flow in a complex field of body forces of variable magnitude, which later goes over into a jet flow (for sufficiently large eccentricity). The flow is twisted with variable pitch and is accelerated with increasing axial and tangential velocity components. The degree of twisting is determined by the angular velocity which varies along the thickness of the running film as well as along the length of the tube. In the exit section of the distributor $\omega = 0$ and in the section where the jet separates ω has its maximum value.

An analysis of the equation of motion for the investigated case permits to obtain the basic criterial parameters characterizing the described flow. Among these parameters are the Reynolds number Re_{cir} determined from the circular velocity:

$$Re_{cir} = \frac{w_{cir} \bar{\delta}}{\nu}, \tag{11}$$

criterion K_{cen} characterizing the effect of the centrifugal forces on the liquid flow:

$$K_{cen} = \left(\frac{\omega R}{w} \right)^2 \tag{12}$$

and criterion K_{cor} taking account of the effect of coriolis forces:

$$K_{cor} = \frac{\omega^2 R^4}{\nu^2}. \tag{13}$$

The degree of twisting of the film is given by the formula [7]

$$C = \frac{\frac{1}{2} \frac{w_{max}}{w_{max}}}{1 - \frac{1}{4} \left(\frac{w_{max}}{w_{max}} \right)^2}. \tag{14}$$

In view of the fact that $w_r = f(x)$, C is also a function of x along which the overall motion of the flow occurs.

NOTATION

C	is the degree of twisting of the film;
e	is the eccentricity of the distributor slit, m;
$\bar{f}$	is the acceleration vector of the liquid due to gravitational forces, m/sec^2;
f_0	is the area of the distributor slit in plane, m^2;
G_{min} and G_{opt}	are the minimum and optimum flow rates of liquid from 1/8 of the wetted perimeter, m^3/sec;
G_{ve}	is the volumetric flow rate of the liquid through the eccentric slit, m^3/sec;
$g = 9.81\ m/sec^2$	is the acceleration due to gravity;
H	is the height of the static thrust of the liquid in the distributor, m;
h	is the pitch of the spiral line, m;
m	is the power index;
n	is the number of wetted tubes;
P_σ	is the capillary pressure, N/m^2;
$\bar{R}$	is the radius vector, m;
R_{tr}	is the wetted radius of the tube, m;
S	is the mean width of the distributor slit, m;
$\bar{w}$	is the velocity vector, m/sec;
$\bar{w}_a$	is the absolute velocity vector of the flow, m/sec;
w_{max}	is the maximum velocity of the flow, m/sec;
w_{rmax}	is the maximum radial velocity, m/sec;
w_{cir}	is the circular velocity of the flow, m/sec;
x	is the coordinate axis directed vertically downward;
$\varkappa$	is the proportionality coefficient;
μ	is the flow rate coefficient for concentric slit;
ν	is the kinematic viscosity coefficient, m^2/sec;
ρ	is the density of the liquid, kg/m^2;
τ	is the time, sec;
φ	is the angle of rotation of the jet, deg;
$\bar{\omega}$	is the angular velocity of the main liquid flow, 1/sec.

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